SYMMETRY BREAKING IN STRING THEORY

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Abstract

In this paper we discuss symmetry breaking in string theory. Spacetime symmetries are implemented as inner automorphisms of the underlying superconformal algebra. Conserved currents generate unbroken spacetime symmetries. As we deform the classical solutions of the string equations of motion, the deformed currents continue to generate spontaneously broken symmetries eventhough they cease to commute with the string Hamiltonian. We illustrate these ideas by studying supersymmetry breaking in a non-trivial string background.

One of the outstanding problems of string theory is to understand the symmetry principles underlying the theory. This is a non-trivial problem because, unlike most physical theories, string theory has been formulated from the beginning as a set of rules for calculating scattering amplitudes. The problem is akin to discovering the full (spontaneously broken) $SU(2) \times U(1)$ gauge invariance of electroweak interactions from the Feynmann rules of the Standard Model.

There are persuasive reasons to believe that string theory does possess a much richer symmetry structure and it is surely important to understand it. The interactions are unique, presumably fixed by symmetry, scattering amplitudes exhibit universal behaviour [1] and the discovery of an infinite symmetry algebra for the bosonic string theory [2] strongly suggest the existence of an enormous underlying symmetry.

A general formalism for determining the spacetime symmetries of string theory was presented in [3]. This formalism relates deformations of the BRST operator to spacetime symmetries.

To each solution of the string equations of motion corresponds a superconformally invariant two-dimensional field theory. This theory will be completely defined by specifying its BRST operator as a local function of world-sheet fields and their canonical momenta. The spacetime fields appear as the couplings of this two-dimensional field theory. Thus, in string theory, the BRST operator Q_{Φ} is parameterized by spacetime fields Φ .

If there is a transformation of the spacetime fields that is a symmetry of string theory, then to every solution of the equations of motion there will be a new solution, where the fields take their transformed values. Thus to every superconformal field theory there will

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correspond another, with transformed couplings. Furthermore, since they are related by a symmetry, these two solutions should be physically indistinguishable. Two superconformal field theories are isomorphic if they are isomorphic as operator algebras, that is if there is a map

$$\rho: \mathcal{A}_1 \mapsto \mathcal{A}_2 \tag{1}$$

between the operator algebras \mathcal{A}_1 and \mathcal{A}_2 of the two superconformal field theories that maps the BRST operator of the one theory to the BRST operator of the other and preserves the equal time commutation relations. For any algebra we may construct another algebra isomorphic to the first one, by means of an infinitesimal similarity transformation, also called inner automorphism. That is, take $\mathcal{A}_1 = \mathcal{A}_2$ and

$$\rho_h(a) = a + i[h, a] \tag{2}$$

where h is any fixed, infinitesimal operator.

For any infinitesimal operator h, then, the superconformal field theories specified by Q_{Φ} and $Q_{\Phi+\delta\Phi}$ are isomorphic. Thus if

$$i[h, Q_{\Phi}] = Q_{\Phi + \delta \Phi} - Q_{\Phi} = \delta Q \tag{3}$$

for some $\delta\Phi$, it follows that $\Phi\mapsto\Phi+\delta\Phi$ is a symmetry of the spacetime fields.

We may clarify the way in which the change in the BRST operator may be interpreted as a change in the spacetime fields by first considering the more general problem of deforming a superconformal field theory. How does the SCFT deform as we deform the classical solution of the string equations of motion [4, 5, 6]? In general this deformation will not correspond to a symmetry transformation, it is a physically different, nearby solution. For example, in General Relativity, two nearby solutions are flat Minkowski space and a weak gravitational wave propagating through it. We seek then deformations of the BRST operator $Q \mapsto Q + \delta Q$ that preserve nilpotency

$$\{Q, \delta Q\} = 0 \tag{4}$$

It is straightforward to show that to first order nilpotency is preserved by deforming the BRST charge with

$$\delta Q = \int d\sigma c V(\sigma) \tag{5}$$

where $V(\sigma)$ is a BRST invariant operator.

The second question is "What subset of the above deformations do correspond to symmetry transformations?". If we take the generator h to be the zero mode of an infinitesimal $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ superprimary field, then it is straightforward to see that its action on the BRST charge is necessarily of the form of Eq. (5) and thus can be interpreted as a change in the spacetime fields.

It is well known that conserved currents [7, 8] generate symmetries, but within the formalism described here, conservation is not necessary, a fact that does not seem to have been widely appreciated. Indeed it is not hard to demonstrate that a non-conserved

current generates a spacetime symmetry that is spontaneously broken by the particular background.

In the remaining of this paper we shall discuss how supersymmetry is implemented as an inner automorphism of the operator algebra. Initially we shall consider string propagation on $M^7 \times T^3$, where M^7 is 7-dimensional Minkowski spacetime and T^3 a 3-dimensional torus. Spacetime supersymmetry is generated by a conserved two-dimensional current. As we deform our classical solution by turning on a constant H = dB flux along the toroidal dimensions-shifting the vacuum-spacetime supersymmetry is spontaneously broken and is implemented by a non-conserved two dimensional current.

The operator that generates 7-dimensional supersymmetry transformations about the background $M^7 \times T^3$ in the heterotic string is [9]

$$h = \int d\sigma \epsilon^{\alpha a}(X) S_{\alpha a} e^{-\frac{\phi}{2}}(\sigma) = \int d\sigma \epsilon^{\alpha a}(X) J_{\alpha a}(\sigma)$$
 (6)

where $S^{\alpha a}$, $e^{-\frac{\phi}{2}}$, are the spin fields for the two-dimensional fermions $\psi_{\mu}(\sigma)$, and the superconformal ghosts $\beta(\sigma)$, $\gamma(\sigma)$ respectively. We have written the operators in a 7-dimensional notation, $\alpha=1,\cdots 8$ are the SO(7) spinor indices and a=1,2 an internal index. We shall also use the following decomposition of the Dirac matrices

$$\Gamma^{\mu} = \frac{i}{\sqrt{2}} (\gamma^{\mu} \otimes 1 \otimes \sigma^{1}), \qquad \Gamma^{i} = \frac{i}{\sqrt{2}} (1 \otimes \sigma^{i} \otimes \sigma^{2})$$
 (7)

where γ^{μ} satisfy the Clifford algebra in 7 dimensions $\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}$ and σ^{i} are the Pauli matrices.

The currents $J_{\alpha a}(\sigma)$ are conserved $[H, J_{\alpha a}(\sigma)] = 0$, where H is the two-dimensional string Hamiltonian. The integrand is superprimary of dimension $(\frac{1}{2}, 0)$ only if the parameters of transformation $\epsilon^{\alpha a}(X)$ satisfy

$$\gamma^{\mu}\partial_{\mu}\epsilon^{\alpha a} = 0. \tag{8}$$

Let's calculate then the commutator of Q with h given by Eq. (6). We find

$$\delta Q = i[h, Q] = \int d\sigma c \partial_{\lambda} \epsilon^{\alpha a} S_{\alpha a} e^{-\frac{\phi}{2}} \overline{\partial} X^{\lambda}. \tag{9}$$

A spacetime symmetry is unbroken when the vacuum values of the fields are invariant under the transformation. In our formalism this would imply that $\delta Q=0$ and this corresponds to the condition that a spacetime symmetry is unbroken by the particular string background. What would then this imply for supersymmetry in M^7 ? We observe that this condition is satisfied only if the parameter of supersymmetry transformations ϵ^{α} is constant. This confirms then the well known result that global N=2 supersymmetry is unbroken in the string background $M^7 \times T^3$.

Next we deform our classical solution by turning a constant H_{ijk} flux along the three compact dimensions of T^3 . Thus our background consists of $M^7 \times T^3$ and a nonzero flux. This nontrivial string background is described in terms of a nilpotent BRST operator $\hat{Q} = Q + \delta Q$, where Q is the BRST operator that describes string propagation on $M^7 \times T^3$ and δQ the nilpotent deformation that describes the nontrivial H_{ijk} flux [10],

$$\delta Q = \int d\sigma (\frac{1}{2}cB_{ij}\partial X^i \overline{\partial} X^j + \frac{1}{2}c\partial_k B_{ij}\psi^k \psi^i \overline{\partial} X^j - \frac{1}{4}\gamma B_{ij}\psi^i \overline{\partial} X^j)(\sigma). \tag{10}$$

The generator of supersymmetry transformations in the particular background becomes

$$\hat{h} = \int d\sigma \epsilon^{\alpha a} (S_{\alpha a} e^{-\frac{\phi}{2}} + \frac{1}{4} \sigma^{ij} B_{ij} S_{\alpha a} e^{-\frac{\phi}{2}})(\sigma) = \int d\sigma \epsilon^{\alpha a} \hat{J}_{\alpha a}(\sigma). \tag{11}$$

The deformed supersymmetry currents $\hat{J}_{\alpha a}$ cease to be conserved with respect to the deformed Hamiltonian, $[\hat{H}, \hat{J}_{\alpha a}] \neq 0$ but continue to generate supersymmetry transformations about the particular background. The integrand remains a superprimary field of dimension $(\frac{1}{2}, 0)$ if the parameters of transformation obey the following conditions

$$\gamma^{\rho}\partial_{\rho}\epsilon^{\alpha a} + \frac{m}{2}\epsilon^{\alpha a} = 0, \tag{12}$$

where $H_{ijk} = m\epsilon_{ijk}$.

Furthermore we can calculate the commutator of \hat{Q} with \hat{h} , Eq. (11). The result is

$$\delta Q = i[\hat{h}, \hat{Q}] = \partial_{\lambda} \epsilon^{\alpha a} S_{\alpha a} e^{-\frac{\phi}{2}} \overline{\partial} X^{\lambda} - \frac{im}{2} \sigma_{i} \epsilon^{\alpha a} S_{\alpha a} e^{-\frac{\phi}{2}} \overline{\partial} X^{i} + \frac{1}{4} \partial_{\lambda} \epsilon^{\alpha a} \sigma^{ij} B_{ij} S_{\alpha a} e^{-\frac{\phi}{2}} \overline{\partial} X^{\lambda}$$

$$(13)$$

We observe that it is imposible to have $\delta Q = 0$ for any value of the parameters of transformation ϵ^{α} . This indicates that N = 2 supersymmetry is spontaneously broken by the non-zero vacuum value of the H flux. Furthermore this formalism can be used to describe in detail both the Higgs and the SuperHiggs mechanisms in string theory [11], [12].

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